

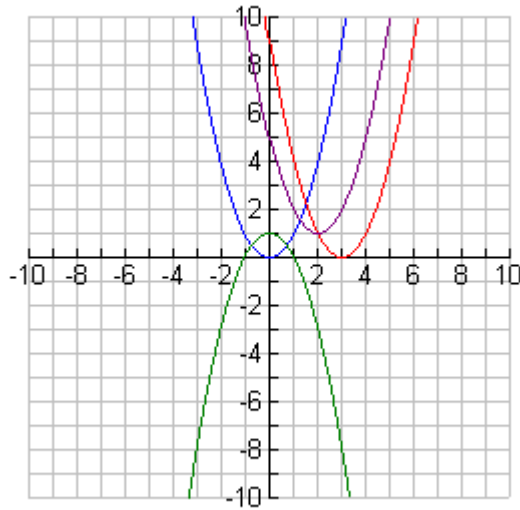
### 3.5 Transformations

$$y = x^2$$

$$y = (x-3)^2$$

$$y = 1 - x^2$$

$$y = x^2 - 4x + 5$$



**Transformations** – functions that map real numbers to real numbers

**Rigid Transformations** – leave the size and shape of a graph unchanged (horizontal and vertical translations, reflections)

**Non-rigid transformations** – distort shape of graph (horizontal and vertical stretches and shrinks)

#### **Vertical Translations:**

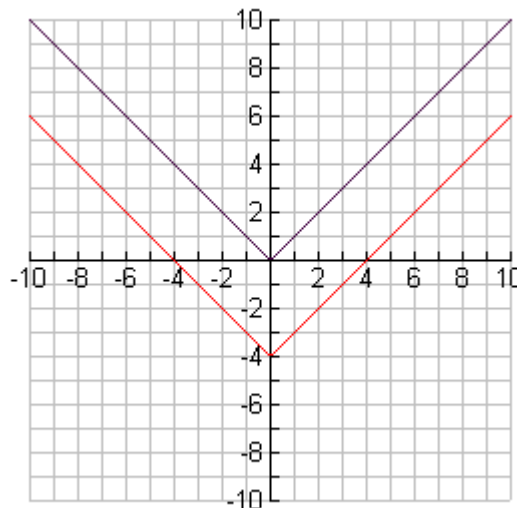
Vertical translation shifts graph up or down.

$y = f(x) + k$  up by  $k$  units.

$y = f(x) - k$  down by  $k$  units

Ex.  $y = |x|$

$$y = |x| - 4$$



## Horizontal Translations:

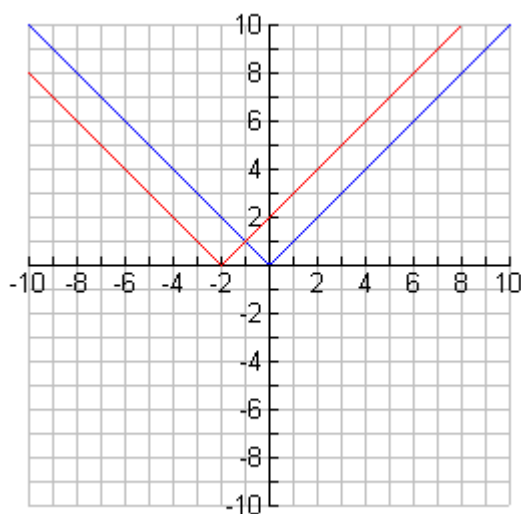
Horizontal translation shifts graph left or right.

$y = f(x - h)$  to right by  $h$  units.

$y = f(x + h)$  to left by  $h$  units.

$$y = |x|$$

$$y = |x + 2|$$



## Reflections:

Points  $(x, y)$  and  $(x, -y)$  are **reflections of each other across the x-axis**.

Across the x-axis:  $y = f(x) \rightarrow y = -f(x)$

Points  $(x, y)$  and  $(-x, y)$  are **reflections of each other across the y-axis**.

Across the y-axis:  $y = f(x) \rightarrow y = f(-x)$

## Finding Equations of Reflections:

Find an equation for the reflection of  $f(x) = \frac{3x-7}{x^2+2}$  across each axis.

Reflection across x-axis:  $y = -f(x)$

$$-f(x) = -\left(\frac{3x-7}{x^2+2}\right) = \frac{-3x+7}{x^2+2}$$

Reflections across y-axis:  $y = f(-x)$

$$f(-x) = -\left(\frac{3(-x)-7}{(-x)^2+2}\right) = \frac{-3x-7}{x^2+2}$$

## Stretches and Shrinks

Let  $a$  or  $b$  be a positive real number. Then the following transformations result in stretches or shrinks of the graph of  $y = f(x)$ .

### Horizontal Stretches or Shrinks

$y = f\left(\frac{x}{b}\right)$  a stretch by a factor of  $b$  if  $b > 1$

a shrink by a factor of  $b$  if  $0 < b < 1$

### Vertical Stretches or Shrinks

$y = a \cdot f(x)$  a stretch by a factor of  $a$  if  $a > 1$

a shrink by a factor of  $a$  if  $0 < a < 1$

### Examples:

Transform the given function by a) a **vertical stretch** by a factor of 3, and b) a **horizontal shrink** by a factor of  $1/2$ .

$$f(x) = |x + 4|$$

$$\text{a) } 3f(x) = 3|x + 4|$$

$$\text{b) } f\left(\frac{x}{1/2}\right) = f(2x) = |2x + 4|$$

### Sequence of transformations:

1) Reflections, 2) Stretches and shrinks and 3) shifts

Describe a basic graph and a sequence of transformations that can be used to produce a graph of the given function.

$$y = -2\sqrt{x+3}$$

Starting with  $y = \sqrt{x}$ , reflect across the x-axis, vertical stretch by factor of 2, and shift left 3.

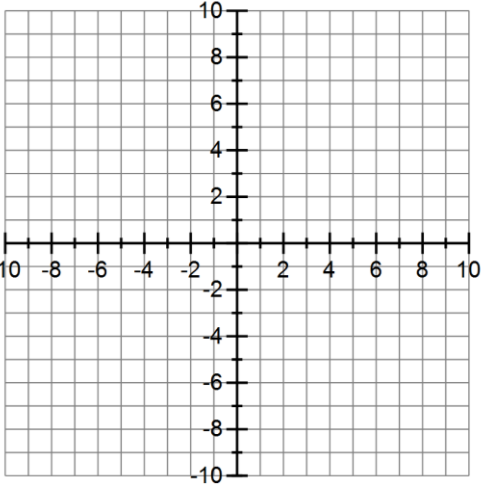
A new graph is obtained from the series of transformations on the given graph; write the equation for the new graph.

$y = x^2$ ; a vertical stretch by a factor of 4, then a shift right 6 units.

$y = 4(x - 6)^2$

**Examples:** Graph the following:

a)  $f(x) = (x - 1)^3 + 2$



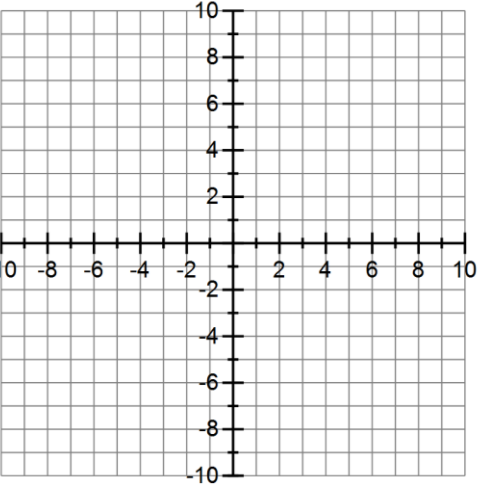
Parent function:

Parent table:

x	y

x	y

b)  $g(x) = 2|x + 1| - 3$



Parent function:

Parent table:

x	y

x	y